## Section 6.4

## Permutations

A permutation of $n$ items is an ordered list of those items. The number of possible permutations of $n$ items is given by $n$ factorial, which is

$$
n!=n \times(n-1) \times(n-2) \times \cdots \times 2 \times 1
$$

for $n$ a positive integer, and $0!=1$.

## Permutations of $\boldsymbol{n}$ items taken $\boldsymbol{r}$ at a time

A permutation of $n$ items taken $r$ at a time is an ordered list of $r$ items chosen from a set of $n$ items. The number of permutations of $n$ items taken $r$ at a time is given by

$$
P(n, r)=\frac{n!}{(n-r)!}=n \times(n-1) \times(n-2) \times \cdots \times(n-r+1)
$$

## Combinations of $\boldsymbol{n}$ items taken $\boldsymbol{r}$ at a time

The number of combinations of $n$ items taken $r$ at a time is given by

$$
C(n, r)=\frac{P(n, r)}{r!}=\frac{n!}{r!(n-r)!}=\frac{n \times(n-1) \times(n-2) \times \cdots \times(n-r+1)}{r!} .
$$

Problem 1. How many ordered sequences are possible that contain four items chosen from six? Solution here.

Problem 2. How many unordered sets are possible that contain three objects chosen from seven? Solution here.

Problem 3. How many five-letter sequences are possible that use the letters $b, o, g, e, y$ once each? Solution here.

Problem 4. How many three-letter (unordered) sets are possible that use the letters $q, u, a, k, e, s$ at most once each?
Solution here.

Problem 5. A bag contains 3 red marbles, 2 green ones, 1 lavender one, 2 yellows, and 2 orange marbles.
a) How many possible sets of four marbles are there?
b) How many sets of three marbles include all the red ones?
c) How many sets of four marbles include none of the red ones?
d) How many sets of four marbles include one of each color other than lavender?
e) How many sets of five marbles include at least two red ones?
f) How many sets of five marbles include either the lavender one or exactly one yellow but not both colors?

Solution here.

Problem 6. Ben and Ann are among 7 contestants from which 4 semifinalists are to be selected. Of the different possible selections, how many contain Ben but not Ann?

Solution here.

Problem 7. If 10 persons met at a reunion and each person shakes hands exactly once with each of the others, what is the total number of handshakes?

Solution here.

